Economic MPC and Real-time Decision Making with Application to Large-Scale HVAC Energy Systems

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1. Stanford Energy System Innovations (SESI) project

2. Expanding the Industrial Scope of Model Predictive Control
   - Discrete decisions
   - Is the control robust?
   - Economic MPC with periodic constraint

3. Conclusions

4. Future Work
The $485-million Stanford Energy System Innovations (SESI) project; replaced an aging 50-MW natural-gas-fired cogeneration plant with a new heat-recovery system to provide heating and cooling to the campus.
A new 80-megavolt-ampere electrical substation brings electricity from the grid. Crews also converted 155 campus buildings from steam to hot-water distribution and installed a 22-mile-long network of new pipe.
The star of the show: three heat-recovery chillers—the largest in the U.S.—that strip waste heat from 155 campus buildings.
Johnson Controls developed the Central Energy Plant Optimization Model (CEPOM); the algorithm optimizes a 10-day forecast every 15 minutes, considering campus loads, weather patterns, price of electricity, available equipment and many other factors.
Large-scale commercial application
Control Decomposition

High Level

- Disturbance Forecasts, Electricity Pricing
  - Cooling Load
  - Aggregate System Curve
- Demand Profile

Low-Level Airside
- Disturbance Estimate
- Measurements
- Temperature Setpoints

Airside Subsystem 1
- Airside PID 1

Airside Subsystem 2
- Airside PID 2

Airside Subsystem n
- Airside PID n

Low-Level Waterside
- Measurements
- Equipment Setpoints
- Waterside PID

Waterside Subsystem
The disturbance forecast: weather and electricity prices

Amb. Temp. (°C)

Elec. price ($/kWh)
High-level problem: Optimal production and average building temperatures (25 buildings)
Low level airside: Optimal zone temperatures and setpoints (20 zones in each of 25 buildings)
Low level waterside: Production and Gantt chart for central plant equipment
The optimizations were solved using Gurobi 6.0 via MATLAB R2016b on a machine with 8GB RAM and 2.66GHz Intel Core 2 Quad Processor Q8400.

- The high-level problem took 35 seconds to solve.
- The low-level airside subproblems took about 15 seconds each to solve.
- The low-level waterside subproblem was given two minutes of computation time, after which the incumbent solution (with an optimality gap of 0.2%) was accepted.
- Since control executions occur every 15 minutes, this decomposition can easily be implemented online.
- Solution times can be further decreased by using a horizon shorter than one week (Risbeck, Maravelias, Rawlings, and Turney, 2016).
In operation since December 2015 (Wenzel, Turney, and Drees, 2016).

The central plant was run in autonomous mode about 90% of the time (including time off-line for plant maintenance).

Achieved 10% to 15% additional savings in operating costs compared to control by the best team of trained human operators (Stagner, 2016).

This large-scale implementation demonstrates the significant potential benefits to applying model-based optimization to large HVAC systems.
Literature review for multiple problem elements

**Buildings**

**Stochastic MPC for buildings**
- Oldewurtel, Parisio, Jones, Gyalistras, Gwerder, Stauch, Lehmann, and Morari (2012)
- Ma, Matuško, and Borrelli (2015)

**Scheduling for central plants with TES**
- Kapoor, Powell, Cole, Kim, and Edgar (2013)

**Scheduling/control for TES and buildings**
- Touretzky and Baldea (2016)

**Discrete Actuators**

**Mixed logical dynamical/piecewise affine systems**
- Bemporad and Morari (1999)
- Lazar, Heemels, Weiland, and Bemporad (2006)

**Switched Systems**

**Quantization as a disturbance**
- Quevedo, Goodwin, and De Doná (2004)
- Kobayshi and Hiraishi (2013)

**Economic MPC**

**Stability**
- Diehl, Amrit, and Rawlings (2011)
- Ellis, Durand, and Christofides (2014)
- Grüne and Stieler (2014)

**Average Performance**
- Angeli, Amrit, and Rawlings (2012)
- Müller, Angeli, and Allgöwer (2014)

**Periodic Systems**

**Linear**
- Böhm, Raff, Reble, and Allgöwer (2009)
- Limon, Alamo, de la Peña, Zeilinger, Jones, and Pereira (2012)

**Nonlinear**
- Huang, Harinath, and Biegler (2011)
- Zanon, Gros, and Diehl (2013)
- Falugi and Mayne (2013)
Problem 1—stabilizing a steady state

Stability Assumption: $V_f(f(x, \kappa_f(x))) \leq V_f(x) - \ell(x, \kappa_f(x))$
Denoting the (possibly multivalued) control law as $\kappa_N(\cdot)$, closed-loop system is

$$x^+ \in F(x) := \{ f(x, u) \mid u \in \kappa_N(x) \}$$  \hspace{1cm} (1)

**Theorem 1 (Exponential stability of (sub)optimal MPC)**

The origin of the closed-loop system (1) is exponentially stable on (arbitrarily large) compact subsets of the feasible set $\mathcal{X}_N$. (Pannocchia, Rawlings, and Wright, 2011)

*Feasible set $\mathcal{X}_N$ is the set of $x$ that can reach $\mathbb{X}_f$ within $N$ steps while meeting constraints.*
So the first question of interest is how much effort is required to extend all of the existing MPC theory to handle discrete actuators.

The answer, surprisingly, is *none*. Consider the main assumption about the input feasible set (Rao and Rawlings, 1999)

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Notice that $\mathbb{U}$ is not required to contain the origin in its interior as is common in much of the early MPC literature.

Therefore, to treat discrete actuators, we simply change the set $\mathbb{U}$ to enforce discreteness in some subset of the actuators.
Figure 1: Typical input constraint sets $\mathcal{U}$ for (a) continuous actuators and (b) mixed continuous-discrete actuators; the origin (●) is the equilibrium of interest.
Problem 2—Inherent robustness of (sub)optimal MPC

Real systems are affected by disturbances that cause the nominal model to no longer hold

- Process errors $d$ lead to model error $x^+ = f(x, u) + d$
- Measurement errors $e$ corrupt $x$ estimate to $x + e$
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With disturbances, the closed-loop system becomes

$$x^+ \in F_{de}(x) := \{f(x, u) + d \mid u \in \kappa_N(x + e)\}$$ (2)

Theorem 2 (Robust exponential stability of (sub)optimal MPC)

The origin of the perturbed closed-loop system (2) is robustly exponentially stable on (arbitrarily large) compact subsets of the feasible set $X_N$. (Pannocchia et al., 2011)
Once again, the extension to discrete actuators is immediate.

- The set $\mathcal{U}$ need not be convex, connected, etc.—it need only contain the origin.
Problem 2—Extension to discrete actuators

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However, design choices become more striking with discrete actuators:
- Theory precludes a large change in control action at the setpoint
  - System must be locally stabilizable using only unsaturated actuators
  - Discrete actuators are always saturated
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- Single (set)point stabilization may no longer be an appropriate goal
MPC is stabilizing on $\mathcal{X}_N$
Feasible Sets

- MPC is stabilizing on $\mathcal{X}_N$ but $\mathcal{X}_N$ may not be what you expect.
Let the sequences \((x_p, u_p)\) denote a given \(T\)-periodic solution to a periodic nonlinear system

\[
\begin{align*}
    x_p(i + 1) &= f(x_p(i), u_p(i), i) \\
    x_p(i + T) &= x_p(i), \quad u_p(i + T) = u_p(i)
\end{align*}
\]

- In the third problem, we assume a periodic solution is available, but change the controller's goal from stabilization of the periodic solution (tracking) to optimization of economic performance.
- The periodic solution then serves as a useful end constraint for the economic optimization problem.
- The stage cost \(\ell(x, u)\) is free to be chosen as an economic profit function and has no connection to distance from \((x_p, u_p)\) as in the tracking case.
Setup for economic MPC with periodic constraint

Figure 2: The periodic solution $x_p(t)$ as end constraint for economic MPC problem for a system with initial condition $(x, t)$ and $N = 2$. 
Mixed-Integer Programming

Mixed-integer optimization has the following general form:

\[
\min_{x \in \mathbb{R}^n} \quad f(x)
\]

\[\text{s.t.} \quad g(x) \leq 0\]

\[x_i \in \mathbb{I}, \quad i \in \mathbb{I} \subseteq \{1, \ldots, n\}\]

- Objective function \(f\), constraints \(g\), and integer variables \(\mathbb{I}\)
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  - \(g\) is linear (i.e., affine)
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subject to

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MIQP software (e.g., Gurobi, SCIP, CPLEX) can find solutions much more efficiently than a brute force search.

- Relax integrality constraints and solve
- Branch on fractional variables to enforce integrality and repeat
Computational Burden

Exhaustive branching can take an extremely long time. Luckily, powerful solvers have a number of other techniques.

- Pre-solve to remove unnecessary variables or equality constraints
- Derive valid inequalities to improve relaxation bound
- Employ strong branching or other methods to choose the “best” fractional variable to branch on
- Retain basis information so re-solving a branch is quick
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However, MIQP is attractive for a number of reasons.

- Discrete variables can model discrete (e.g., on/off) decisions
- A conservative estimate of optimality gap is available
- Less difficult than general nonlinear global optimization (with or without discrete variables)
Example: Simplified Building Cooling

\[
\frac{dT}{dt} = -k(T - T_{amb}) + q_{amb} + q_{ch} - q_{tank}
\]
\[
\frac{ds}{dt} = -\sigma s + q_{tank}
\]

\[
vq_{min} \leq q \leq vq_{max}
\]

\[
q_{tank} \leq q_{ch}, \quad v \in \{0, 1, 2\}
\]

\[
x := (T, s)
\]
\[
u := (q_{ch}, q_{tank}, v)
\]
\[
d := (T_{amb}, q_{amb})
\]

- Temperature must be maintained within preset bounds.
- Each chiller can be on or off.
- When on, chillers have minimum and maximum capacity.
Parameters

Stage Costs

\[ \ell_{\text{econ}}(x, u, t) := \rho(t)q(t) \]
\[ \ell_{\text{track}}(x, u, t) := |x(t) - x_p(t)|_Q^2 + |u(t) - u_p(t)|_R^2 \]

- Horizon \( N = 24 \)
- Periodic \( \rho, T_{\text{amb}}, q_{\text{amb}} \)
- Weights \( Q = R = I \)
Periodic solution can be used for tracking or end constraint.

- Precooling reduces cooling during peak price hours.
- “Bounds” on $q$ are determined by the (integral) value of $v$. 
Tracking MPC converges to the periodic reference.

- Initial condition $T(0) = 2, s(0) = 0$.
- Stage cost penalizes changes in $T$, $s$, $q_{ch}$, $q_{tank}$, and $v$. 

Economic Cost: 77.80
Using the economic objective, cost is reduced by 3.3%.

- Controller aggressively pursues lower cost.
- Deviation in $u$ is not penalized.
MPC is well suited to high-level operational goals like energy or cost minimization.
Conclusions

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- MPC of large-scale HVAC energy systems offers significant economic benefit over current operations.
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Supporting MPC theory: discrete actuators and general nonlinear systems

- Nominal stability of optimal and suboptimal MPC with a steady-state operating point (Rawlings and Risbeck, 2016)
- Inherent robustness of suboptimal MPC with mixed continuous/discrete actuators (Allan et al., 2016)
- Asymptotic stability of the periodic tracking problem with terminal region
- Asymptotic stability of economic MPC with periodic end constraint
Future Work

- Release test problem based on Stanford SESI system for the research community to benchmark new control system designs (other decompositions, robust MPC, stochastic MPC, etc.)
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Hierarchical decomposition: quantify the complexity/performance tradeoff

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Combined scheduling and control

More large-scale applications. Move the needle on total US energy consumption
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- Process industries: anything planned besides DMC(n)?
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