Computation and Uncertainty
The Past, Present and Future of Control

Manfred Morari

University of Pennsylvania
United Technologies Research Center

FOCAPO / CPC 2017 - Tuscon, AZ
January 9, 2017
Outline

• Past  
  – Where we came from: A reflection on my roots

• Present  
  – Where we are: Fast MPC

• Future  
  – Where we should be going: Open research areas
Outline

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U. of Minnesota Chemical Engineering 1975 - 1977

George Stephanopoulos

Rutherford "Gus" Aris 1929-2005
Theory-Practice Gap

Main theme of CPC I in 1976

Explosive development of theory had taken place

- Industry did not understand theory
- Academia had no clue about real controller design

Exceptions: Åström, Gilles, Balchen,…
Theory-Practice Gap: Model Uncertainty

- Control Objective did not address robustness / uncertainty directly. Indirect effect of tuning parameters was not understood (Horowitz, Shinnar, Doyle,...)


Design of Sampled Data Controllers

Zalman J. Paimor$^1$ and Reuel Shinnar$^*$

Department of Chemical Engineering, The City College of The City University of New York, New York, New York 10031

linearized models. A good design procedure must take into account that there is a finite but unknown deviation between the model used for design and the real description of the process. This also applies to probabilistic models of the disturbance.

5. The Controller Must Be Reasonably Insensitive to Changes in System Parameters. It must be stable and perform well over a reasonable range of system parameters.
A turning point...

- IFAC Workshop on Robust Control Systems, Interlaken, Switzerland, October 4-7, 1982. org. by J. Ackermann
- Participants: Barmish, Doyle, Frank, Kwakernaak, Looze, Mansour, Morari, Olbrot, Stein, Toedtli,...
Robust Control of Ill-Conditioned Plants: High-Purity Distillation

SIGURD SKOGESTAD, MANFRED MORARI, MEMBER, IEEE, AND JOHN C. DOYLE

S. Skogestad is with the Department of Chemical Engineering, Norwegian Institute of Technology, Trondheim, Norway.
M. Morari and J. C. Doyle are with the Department of Chemical Engineering, California Institute of Technology, Pasadena, CA 91125.

Abstract—Ill-conditioned plants are generally believed to be difficult to control. Using a high-purity distillation column as an example, the physical reason for the poor conditioning and its implications on control system design and performance are explained. It is shown that an acceptable performance/robustness trade-off cannot be obtained by simple loop-shaping techniques (via singular values) and that a good understanding of the model uncertainty is essential for robust control system design. Physically motivated uncertainty descriptions (actuator uncertainty) are translated into the $H_{\infty}$/structured singular value framework, which is demonstrated to be a powerful tool to analyze and understand the complex phenomena.
Computational Complexity of $\mu$ Calculation

Richard P. Braatz, Peter M. Young,
John C. Doyle, and Manfred Morari

Abstract—The structured singular value $\mu$ measures the robustness of uncertain systems. Numerous researchers over the last decade have worked on developing efficient methods for computing $\mu$. This paper considers the complexity of calculating $\mu$ with general mixed real/complex uncertainty in the framework of combinatorial complexity theory. In particular, it is proved that the $\mu$ recognition problem with either pure real or mixed real/complex uncertainty is NP-hard. This strongly suggests that it is futile to pursue exact methods for calculating $\mu$ of general systems with pure real or mixed uncertainty for other than small problems.
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Model Predictive Control

- Determine state $x(t)$
- Determine optimal sequence of inputs over horizon
- Implement first input $u(t)$
- Wait for next sampling time; $t := t + 1$
As you know, I have been working on my PhD dissertation in the area of Process Control. Recently I implemented one of the techniques from my dissertation research on the Cal Cracker CO Furnace. The technique is working exceedingly well and substantiates the theory on which my dissertation is based. I would like to obtain permission from the company to publish this data in my dissertation. The use of actual process data will give the dissertation credibility and stature.

The data may be presented in dimensionless terms to avoid any association with the actual process or levels of operation. Attached are the data presented in graphical terms, that I would use.

The theory to which I have referred is familiar to you, Egon Deering, Charles Gillard, and Stan Marple in Head Office. However, for others who may read this letter, I have attached a copy of the original research proposal I gave my professor. The technique used on the CO Furnace is the least square approach to reducing the error described in the proposal.

Your consideration of this matter will be appreciated.

C. R. Cutler
# Verifiable Control Synthesis

<table>
<thead>
<tr>
<th>Offline</th>
<th>Online</th>
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<tbody>
<tr>
<td>Explicit MPC</td>
<td>1\textsuperscript{st} Order–Fast Gradient</td>
</tr>
<tr>
<td>Approx. Explicit MPC</td>
<td>Interior Point Opt.</td>
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Verifiable Control Synthesis

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Explicit MPC : Online => Offline Processing

- Optimization problem is parameterized by state
- Control law piecewise affine for linear systems/constraints
- Pre-compute control law as function of state $x$ (parametric optimization)

Result: Online computation dramatically reduced

\[
\begin{align*}
 u^*(x_0) &= \arg\min_{u_i} \sum_{i=0}^{N} l(x_i, u_i) + V_f(x_N) \\
 &\text{s.t. } (x_i, u_i) \in X \times U \\
 &\quad x_{i+1} = f(x_i, u_i) \\
 &\quad x_N \in X_f
\end{align*}
\]

[M.M. Seron, J.A. De Doná and G.C. Goodwin, 2000]
[T.A. Johansen, I. Peterson and O. Slupphaug, 2000]
[A. Bemporad, M. Morari, V. Dua and E.N. Pistokopoulos, 2000]
Verifiable Control Synthesis

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<tr>
<td><strong>Explicit MPC</strong></td>
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<tr>
<td>- &lt; 5 states</td>
<td></td>
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<tr>
<td>- Simple look-up</td>
<td></td>
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<tr>
<td>- &lt; (\mu)s sampling</td>
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| Approx. Explicit MPC | Interior Point |
Verifiable Control Synthesis

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<td>• &lt; μs sampling</td>
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<td><strong>Approx. Explicit MPC</strong></td>
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<td>• &lt; 10 states</td>
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<td>• Specified complexity</td>
<td></td>
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<td>• &lt; μs sampling</td>
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</table>
Computation / Software

Formal specification
- YALMIP
- HYSDEL
- Linear + Hybrid models

Verified controller

Control law
- Explicit MPC
- Fixed-complexity solutions

Software synthesis
- Real-time workshop
- Bounded-time solvers
- Verifiable code generation

Multi-Parametric Toolbox (MPT)
- (Non)-Convex Polytopic Manipulation
- Multi-Parametric Programming
- Control of PWA and LTI systems
- > 32,000 downloads to date

MPT 3.0
### Verifiable Control Synthesis

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<td>• &lt; 5 states</td>
<td>• Any size</td>
</tr>
<tr>
<td>• Simple look-up</td>
<td>• Simple and robust</td>
</tr>
<tr>
<td>• &lt; μs sampling</td>
<td>• μs – ms sampling</td>
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<th><strong>Interior Point</strong></th>
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<tr>
<td>• &lt; 10 states</td>
<td>• Any size</td>
</tr>
<tr>
<td>• Specified complexity</td>
<td>• Highly accurate</td>
</tr>
<tr>
<td>• &lt; μs sampling</td>
<td>• ms sampling</td>
</tr>
</tbody>
</table>
FORCES Pro: Multi-method Autocoder

- From problem & platform specification to implementation
- Generates ANSI-C (C89) code:
  - statically allocated
  - library-free
  - portable to all platforms with C-compiler

\[
\begin{align*}
\min f(x) \\
\text{s.t. } x \in K
\end{align*}
\]

![Diagram showing the process from problem description to embeddable C-code](image)

**FORCES Pro**

- fit method to problem
- exploit problem structure
- tailor code to platform

*Platform specs: ARM, Xilinx*

*embotech*
Supported Problems & Platforms

- Optimized for parametric **multistage** convex programs of the form

\[
\text{minimize} \sum_{i=1}^{N} \frac{1}{2} z_i^T H_i z_i + f_i^T z_i \\
\text{subject to} \quad D_1 z_1 = c_1 \\
C_{i-1} z_{i-1} + D_i z_i = c_i, \\
z_{\text{min}} \leq z_i \leq z_{\text{max}}, \\
A_i z_i \leq b_{\text{max}}, \\
z_i^T Q_{i,k} z_i + L_{i,k} z_i \leq r_{i,k}
\]

- separable objective
- affine equalities, each coupling only two consecutive variables
- upper/lower bounds
- affine inequalities
- quadratic inequalities

**Example Problems**
- Optimal control
- MPC, MHE
- polynomial data fitting

**Interfaces**
- Matlab
- Simulink
- Python
- dSpace

**Methods**
- Primal-dual interior point
- ADMM 1 & 2, custom projections
- Primal (fast) gradient
- Dual (fast) gradient 1

**Platforms**
- x86, x86_64
- Tricore
- PowerPC
- ARM
More than 150 users world wide & across industries in 35 countries
Applications by the Automatic Control Lab

18 ns  Multi-core thermal management (EPFL)  
       [Zanini et al 2010]

10 µs  Voltage source inverters  
       [Mariethoz et al 2008]

20 µs  DC/DC converters (STM)  
       [Mariethoz et al 2008]

25 µs  Direct torque control (ABB)  
       [Papafotiou 2007]

50 µs  AC/DC converters  
       [Richter et al 2010]

5 ms  Electronic throttle control (Ford)  
      [Vasak et al 2006]

20 ms  Traction control (Ford)  
       [Borreli et al 2001]

40 ms  Micro-scale race cars

50 ms  Autonomous vehicle steering (Ford)  
       [Besselmann et al 2008]

500 ms  Energy efficient building control (Siemens)  
        [Oldewurtel et al 2010]
Model predictive control (MPC) for buildings

- weather
- Meteo Service
  - weather predictions
- Building
  - measurements
- Kalman Filter
  - energy costs
  - comfort criteria
  - occupancy prediction
- MPC
  - model
  - optimization
- control inputs
Brightbox Technologies Inc.
MPC for Building Energy Mgt

- Flawless operation in several commercial bldgs.
- Most complex building: 8 packaged units and 600 vav boxes
  - 18,176 signals processed every 5 min.
  - MPC: >300,000 vars. and >500,000 constraints (sampling time 5 mins)
NEXTracker acquires predictive modeling software firm BrightBox

U.S. tracker manufacturer NEXTracker has purchased software firm BrightBox Technologies in a move it says will augment its predictive modeling capabilities and expedite the commissioning process.

NEXTracker, the U.S. firm that builds tracking devices for the solar PV market, has confirmed today the purchase of predictive modeling software company BrightBox.

The acquisition will, according to NEXTracker CEO Dan Shugar, augment the company’s software engineering resources, and brings on board BrightBox co-founders Allan Daly and Francesco Borrelli, who will add their years of modeling and predictive control experience to the NEXTracker team.
Micro-scale Race Cars

- 1:43 scale cars – 106mm
- Top speed: 5 m/s  
  (774 km/h scale speed)
- Full differential steering
- Position-sensing: External vision
- 50 Hz sampling rate

Project goals:
1. Plan optimal path online in dynamic race environment
2. Demonstrate real-time control optimizing car performance
3. Beat all human opponents!

Challenges:
- Interaction with multiple unpredictable opponents
- Highly nonlinear dynamics
- High-speed planning and control
Example: Autonomous Racing

Trajectory optimization 50x/second on smartphone

Example: Autonomous Racing

ORCA - Optimal RC Autonomous Racing

Model Predictive Contouring Control
Dynamic Obstacle Avoidance - Fast

ifa ETH Zürich
Example problem

- Hit back a thrown ball
- Implicit feedback law updated at 20ms
  - Try 10’000 trajectories
    - Sample different ways to hit the ball
    - Apply first 20ms of the best one
Evaluation

- Algorithm evaluated in the Flying Machine Arena

- System limits
  \[ c_{\text{min}} = 5 \text{m/s}^2 \]
  \[ c_{\text{max}} = 20 \text{m/s}^2 \]
  \[ \omega_{\text{max}} = 25 \text{rad/s} \]
Rapid trajectory generation for quadrocopters
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Some Open Research Areas in Control

• Systems with distributed control

• Systems with discrete decisions and switched systems

• Systems with constraints and uncertainty

• Supervisory control systems
Some Open Research Areas in Control

• Systems with distributed control

• Systems with discrete decisions and switched systems

• Systems with constraints and uncertainty

• Supervisory control systems
PWA Hybrid Models

- Piecewise affine (PWA) systems
- Polyhedral partition of state space
- Affine dynamics on reach region

\[
C_r \triangleq \left\{ \begin{bmatrix} x \\ u \end{bmatrix} \in \mathbb{R}^n \mid H_r \begin{bmatrix} x \\ u \end{bmatrix} \leq K_r \right\}
\]

\[
x(t + 1) = A_r x(t) + B_r u(t) + f_r \quad \text{if} \quad \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \in C_r
\]
## Speedup of software for MILP in 15 years

<table>
<thead>
<tr>
<th>Component</th>
<th>Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Program</td>
<td>x 1000</td>
</tr>
<tr>
<td>Integer Program</td>
<td>x 100 – 1000</td>
</tr>
<tr>
<td>Computers</td>
<td>x 1000</td>
</tr>
<tr>
<td>Overall</td>
<td>x 100 million</td>
</tr>
</tbody>
</table>

### Integer Programming

<table>
<thead>
<tr>
<th>Technique</th>
<th>Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preprocessing</td>
<td>x 2</td>
</tr>
<tr>
<td>Heuristics</td>
<td>x 1.5</td>
</tr>
<tr>
<td>Cutting Planes</td>
<td>x 50</td>
</tr>
</tbody>
</table>

Source: *Bixby, Gu, Rothberg, Wunderlich 2004*
Predictive Control
Applications and Strategies
MIP in power electronics applications

• New multilevel topologies emerging for high efficiency and power quality

15 independent pairs of switches operated at frequency > 1kHz, Horizon=50

Control:
– 6 capacitor voltages
– 3 motor currents

• Performance improvement requires accounting for binary nature of manipulated variables

• Need fast MIP solver to optimize performance in real-time
Model predictive control: advancing the frontiers
Industry requirements vs available processing power
Kollsnes has a capacity of 143,000,000 cubic meters ($3.8\times10^{10}$ US gal) of natural gas per day.

Two 41.2 MW compressor strings for gas export are now powered by MPC-controlled LCIs.

Kårstø is Europe's biggest export port for natural gas liquids and the third largest in the world.

Three 7.5 MW booster compressors are now powered by MPC-controlled LCI.

First successful ride-through (29.11.2015)
Some Open Research Areas in Control

• Systems with distributed control

• Systems with discrete decisions and switched systems

• Systems with constraints and uncertainty

• Supervisory control systems
A typical Piping & Instrumentation Diagram
Model checking of safety properties for Simulink Models

Avionics distributed control system complexity:
- 10K-250K simulink blocks
- 40k-150K binary raw variables
- Hundred to few thousand bin’s after simplification/abstraction

Automotive single controller complexity:
- 5K-80K simulink blocks
- Few thousand bin’s after simplification/abstraction

FormalSpecsVerifier tool environment (NuSMV)

Source: Alberto Ferrari
Conclusions

• Themes of Uncertainty and Computation
• For implementation MPC is alternative of choice, but open issues:
  – Distributed control
  – Switches (incl supervisory control)
  – Uncertainty
Raff D’Andrea’s PhD students and Post-Docs since moving to ETH

<table>
<thead>
<tr>
<th>Name</th>
<th>Position/Role</th>
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<tbody>
<tr>
<td>Federico Augugliaro</td>
<td>startup</td>
</tr>
<tr>
<td>Mark Muller</td>
<td>assistant professor (Berkeley)</td>
</tr>
<tr>
<td>Philipp Reist</td>
<td>startup</td>
</tr>
<tr>
<td>Luca Gherardi</td>
<td>startup</td>
</tr>
<tr>
<td>Gajamohan Mohanarajah</td>
<td>startup (founder)</td>
</tr>
<tr>
<td>Markus Waibel</td>
<td>startup (founder)</td>
</tr>
<tr>
<td>Markus Hehn</td>
<td>startup (founder)</td>
</tr>
<tr>
<td>Sergei Lupashin</td>
<td>startup (founder)</td>
</tr>
<tr>
<td>Raymond Oung</td>
<td>startup</td>
</tr>
<tr>
<td>Sebastian Trimpe</td>
<td>group leader (Max Planck)</td>
</tr>
<tr>
<td>Angela Schoellig</td>
<td>assistant professor (U. of Toronto)</td>
</tr>
<tr>
<td>Michael Sherback</td>
<td>startup</td>
</tr>
<tr>
<td>Frederic Bourgault</td>
<td>startup</td>
</tr>
<tr>
<td>Guillaume Ducard</td>
<td>assistant professor (U. of Nice)</td>
</tr>
<tr>
<td>Oliver Purwin</td>
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