Modeling of hydraulic fracturing and design of online optimal pumping schedule

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Outline

1. Motivation

2. Introduction
   • What is hydraulic fracturing?
   • Modeling of hydraulic fracturing

3. Background

4. Proposed work
   • Numerical simulation
   • Model reduction techniques
   • Design of model-based pumping schedules

5. Conclusions
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The **shale revolution** has stimulated tremendous production of oil and natural gas in the United States.
What is shale gas?

Natural gas trapped in rock of very low porosity and permeability

Without rock stimulation, production is economically infeasible

From Canadian society for unconventional reservoirs
What was a game changer?

**Horizontal drilling:** the drill bit is made to turn from the vertical to horizontal direction to extract oil/gas that itself runs horizontally.

**Hydraulic fracturing (fracking):** millions of gallons of water, sand and chemicals are pumped underground to break apart the rock and release the gas.
How does fracking affect oil prices?

Fracking has helped boost the shale oil and gas extraction rate from wells

- It helped lowering oil prices on a global scale
- Since 2014, OPEC is playing a high-stakes game of chicken with the United States by bringing supply levels up to lower the oil price

Fracking is at least 10 times more expensive than traditional oil extraction

- If oil prices drop below the break-even price, which is around $50/barrel, the high expense of fracking is no longer justified
- Technological advancements are necessary to make it less costly
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Hydraulic fracturing

(Holditch, 2012)
Hydraulic fracturing: perforation stage

A well is drilled and a wire equipped with explosive charges is dropped into the well.

Explosions along the wellbore are used to create initial fracture paths.
Hydraulic fracturing: dirty volume stage

A fracturing fluid (dirty volume) consisting of water, additives, and proppant (sand) is introduced to further propagate fractures.
Hydraulic fracturing: closure stage

After pumping is stopped and the remaining fluid leaks off to the formation, the natural stress by the formation closes the fracture, trapping the proppant inside the fracture walls.

The trapped proppant forms a highly-conductive channel that helps extracting the oil and gas inside the reservoir effectively.

(Warpinski, 2009)
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Perkins, Kern, and Nordgren (PKN) model

Modeling assumptions:

• The vertical fracture is confined within a single horizontal rock layer

• The rock properties (e.g., Young’s modulus) remain constant over the time and space domains

• The fluid pressure across the vertical direction is constant (because $L \gg W$)

Fracture propagation is a very complex process, involving several co-dependent sub-processes
Modeling of fracture propagation

Fluid flow rate (lubrication theory) for a Newtonian fluid in an elliptical section is highly nonlinear (Nordgren, 1972):

\[ \frac{dP}{dx} = -\frac{64\mu Q}{\pi(H - \delta)W^3} \]

Rock deformation (elasticity equation): The mechanical response of the rock to the loading by fluid pressure (Sneddon et.al, 1946)

\[ W_{max} = \frac{2P(H - \delta)(1 - \nu^2)}{E} \]

\( P \): Net pressure
\( \delta \): Proppant bank height
\( \nu \): Poisson ratio of the formation
\( E \): Young’s modulus of the rock formation
\( \mu \): Fluid viscosity
Modeling of fracture propagation

The conservation of fluid inside the fracture (Nordgren, 1972):

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + U = 0
\]

where \(A = WH\)

\[
Q(0, t) = Q_0 \quad (BC1)
\]
\[
A(L(t), t) = 0 \quad (BC2)
\]
\[
A(x, 0) = 0 \quad (IC)
\]

The fluid leak-off rate is given by (Howard and Fast, 1957):

\[
U(x, t) = \frac{2C_{\text{leak}}}{\sqrt{t - \tau(x)}}
\]

\(C_{\text{leak}}\): Leak-off coefficient
\(\tau(x)\): The fracture-opening time of the location \(x\)
Modeling of proppant transport

Gravity-induced proppant settling velocity in a slurry (Daneshy, 1978)

\[ V_s = \frac{(1 - C)^2}{10^{1.82C}} \left( \frac{\rho_{sd} - \rho_f}{18} \right) g d^2 \]

Viscosity of slurry depends on proppant concentration (Adachi et al., 2007)

\[ \mu = \mu_0 \left(1 - \frac{C}{C_{max}}\right)^{-1.5} \]

- \( g \) : the gravitational acceleration
- \( C \) : proppant concentration
- \( C_{max} \) : saturation concentration
- \( \rho_{sd} \) : proppant particle density
- \( \rho_f \) : fluid density
- \( d \) : proppant diameter
Modeling of proppant transport

**Evolution of proppant bank height** (Gu et al., 2015)

\[
\frac{d(\delta W)}{dt} = \frac{CV_s W}{1 - \phi}
\]

**Advection** of suspended proppant in the horizontal direction (Adachi et al., 2007)

\[
\frac{\partial(CW)}{\partial t} + \nabla \cdot (CWV_p) = 0
\]

where \(V_p\) is the **proppant velocity vector** given by:

\[
V_p = V - (1 - C)V_s
\]

\(\phi\) : porosity of proppant bank
\(V\) : slurry velocity
\(\delta\) : proppant bank height
Propped fracture geometry

- The objective is to achieve uniform proppant bank height

- It is similar to producing uniform proppant concentration at the end of pumping

- A pumping schedule is necessary to achieve this goal.
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Results on proppant pumping schedule design

- A pumping schedule is designed to compensate for fluid leak-off (Nolte, 1986)
- Iteration-based pumping schedule (Gu and Desroches, 2003)
- Proppant transport is not considered for computational efficiency (Dantsov and Pierce, 2014)
- A model-based control approach to achieve the optimal fracture geometry (Gu and Hoo, 2015)
- **Feedback control of proppant concentration at the end of pumping accounting explicitly for proppant transport as well as practical constraints?**
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Challenges in numerical simulation

Coupling nonlinear dynamic equations

• Rock deformation, fluid flow and proppant transport are described by a set of highly coupled nonlinear equations
• Leak-off rate has to be determined via iterations
• Considering proppant settling and transport requires fine meshes

Moving boundary problem:

• The spatial domain changes with time
• The analytical expression for the fracture length is not available
• The number of variables to be determined grows with time
• Particular meshing strategies are required for moving boundary
Numerical solution procedure

We are able to compute the evolution of the fracture length, fracture width, proppant bank height and proppant concentrations in the hydraulic fracturing system.
Simulation results

Full-order solution obtained from an adaptive finite discretization method.

Total number of nodes - 411 spatial & 19611 temporal

Full-order solution for the evolution of the width profile
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Local proper orthogonal decomposition

**Motivation:** dominant spatial patterns change with time due to moving boundary

- Global model reduction technique is no longer applicable
- The eigenfunctions can be tailored to capture the local behavior of a complex nonlinear system more effectively

**Temporally-local model reduction technique**

**Step 1:** Partitioning the time domain into multiple clusters

- Snapshots contained within each cluster exhibit a similar behavior with each other and are relatively dissimilar with the ones contained within the other clusters.
Local proper orthogonal decomposition

Minimization of Euclidean distances between snapshots and the centers of their assigned clusters is formulated as

\[
\begin{align*}
\text{Minimize} & \quad \sum_{c_{ki}, y_{jk}} \sum_{i=1}^{s} \sum_{j=1}^{n} \sum_{k=1}^{m} y_{jk} (x_{ij} - c_{ki})^2 \\
\text{s.t} & \quad \sum_{k=1}^{m} y_{jk} = 1, \quad y_{jk} \in \{0,1\}
\end{align*}
\]

Use **Global Optimum Search (GOS)** algorithm to solve this MINLP problem

- General Bender’s decomposition based clustering algorithm (Tan et. al, 2008)
- It performs favorably over other heuristic algorithms including k-means

**Step 2:** Apply Proper Orthogonal Decomposition (POD) locally to each cluster and compute low-dimensional eigenfunctions

**Step 3:** Derive low-dimensional ODE systems using Galerkin’s method
Results: optimal number of clusters

The GOS algorithm predicts the optimal number of clusters by minimizing the intra-cluster sums and at the same time, maximizing the inter-cluster sums.

Intra-cluster and Inter-cluster error sum plots

Clustering balance curve

**Clustering balance** = \((\text{Intra-cluster error sum} + \text{Inter-cluster error sum}) / 2\)
Results: comparison of width profiles

Temporally-local method
(16 clusters with 2 eigenfunctions each)

Global method
(1 cluster with 16 eigenfunctions)

Temporally-local POD is more accurate and computationally efficient in approximating the original nonlinear system with fewer eigenfunctions, compared to global POD.
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Nolte’s pumping schedule (benchmark)

- One of the most widely used pumping schedules (Nolte, 1986)
- It requires efficiency ($\eta$), total pumping time ($t_e$) and pad time ($t_p$)
- The flow rate is fixed and only the concentration is varied
- Increasing the concentration compensates for leak-off

\[
C_{Nolte}(t) = C_{\text{target}} \left[ \frac{t - t_p}{t_e - t_p} \right]^\frac{1-\eta}{1+\eta}
\]

- $t_e$ : total pumping time
- $t_p$ : pad time
- $\eta$ : final fluid efficiency
Concentration profile at the end of pumping

Limitations of Nolte’s pumping schedule

- Both practical constraints and proppant settling are not considered
- Plant-model mismatch may lead to premature termination ($C > C_{\text{max}}$)
  - Fracture with short propped length
- Open-loop operation
Proppant concentration control in fracking

- **Hydraulic fracturing of unconventional/conventional reservoirs**
- **Manipulated input variables**: concentration at the wellbore
- **Real-time measurements**: fracture width at the wellbore and fracture length
- **Control objective**: produce uniform proppant concentration in the horizontal direction at the end of pumping subject to constraints on operating conditions
Desired fracture geometry

Unified fracture design (UFD) is an offline optimization-based technique to determine a set of fracture width and length values that maximizes the productivity of a fractured well (Daal and Economides, 2006)

\[ L_{opt} = \left( \frac{k_f V_p}{2C_{fd}kh} \right)^{0.5} \]
\[ and \ W_{opt} = \left( \frac{C_{fd}kV_p}{2k_f h} \right)^{0.5} \]

Target concentration is obtained by

\[ C_{target} = \frac{M_{prop}}{L_{opt}W_{opt}H} \]

\( k_f \) : proppant permeability  \( C_{fd} \) : fracture conductivity

\( k \) : rock permeability  \( h \) : pay zone height

\( V_p \) : pay zone volume  \( M_{prop} \) : the mass of proppant to be injected
State-space model validation

**State-space model** is used to compute the evolution of the fracture length, average width and proppant concentrations in the hydraulic fracturing system.

The approximate model can be further improved by using nonlinear models.
Optimal pumping schedule design

Optimization problem

\[
\begin{align*}
\min_{C_{\text{stage},j}} & \quad \sum_{i=1}^{5} (C_i(t_f) - C_{\text{target}})^2 \\
\text{s.t} & \quad C_{\text{min}} < C_i(t) < C_{\text{max}} \quad \text{State constraint} \\
& \quad C_{\text{stage},j} \leq C_{\text{stage},j+1} \quad \text{Input constraint} \\
& \quad \sum_{j=1}^{10} 2Q_{\text{stage},j} C_{\text{stage},j} \Delta t_{\text{stage},j} = M_{\text{total}} \quad \text{Material constraint} \\
& \quad L(t_f) = L_{\text{opt}} \quad \text{Fracture geometry constraint} \\
& \quad \dot{x} = Ax(t) + Bu(t - \theta_i) \quad \text{State-space model} \\
& \quad y = Cx(t) \\
& \quad i = 1, \ldots, 5 \quad \text{and} \quad j = 1, \ldots, 10
\end{align*}
\]
Optimal pumping schedule design

Optimization problem

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\begin{align*}
\min_{c_{stage,j}} & \quad \sum_{i=1}^{5} (C_i(t_f) - C_{target})^2 \\
\text{s. t} & \quad C_{min} < C_i(t) < C_{max} \quad \text{State constraint} \\
& \quad c_{stage,j} \leq c_{stage,j+1} \quad \text{Input constraint} \\
& \quad \sum_{j=1}^{10} 2Q_{stage,j}c_{stage,j} \Delta t_{stage,j} = M_{total} \quad \text{Material constraint} \\
& \quad L(t_f) = L_{opt} \quad \text{Fracture geometry constraint} \\
\end{align*}
\]

\[
\begin{align*}
\dot{x} & = Ax(t) + Bu(t - \theta_i) \\
y & = Cx(t) \\
i & = 1, \ldots, 5 \quad \text{and} \quad j = 1, \ldots, 10 \\
\end{align*}
\]

The first value, \( c_{stage,j} \), will be applied to the hydraulic fracturing system until the next sampling time.
Closed-loop system under MPC

- Hydraulic fracture simulator is used to simulate the process
- Real-time measurements of the fracture width at the wellbore and length are used
- A Luenberger observer is used to estimate the proppant concentration inside the fracture
- Reduced-order model is used in the MPC
Input proppant concentration at the wellbore

The generated pumping schedule is the **projection of the Nolte's pumping schedule onto the feasible set** constructed by considering operational/practical constraints.
Concentration profile at the end of pumping

The spatial concentration obtained by the proposed method at the end of pumping is closer to the target concentration compared to Nolte’s pumping schedule.
Novelty of the proposed pumping schedule

- **Proppant transport** and its influence on fracture propagation is considered.
- **Practical constraints** are considered for optimality and safety.
- **Model mismatch can be improved** by using the real-time measurement as a feedback (closed-loop operation).
- Luenberger observer is used to estimate the unmeasurable state such as proppant concentration inside the fracture.
- Reduced-order models are used to reduce the computational requirement to solve the optimization problem.

The proposed pumping schedule is able to overcome the limitations of the state-of-the-art pumping schedules.
\[ \min \sum_{i=1}^{5} (C_i(t_f) - C_{target})^2 \]

s. t. \( C_{min} < C_i(t) < C_{max} \)
\( C_{stage,j} \leq C_{stage,j+1} \)
\( \sum_{j=1}^{10} 2Q_{stage,j}C_{stage,j}\Delta t_{stage,j} = M_{total} \)
\( \tau_i \frac{dC_i}{dt} + C_i = K_i C_{stage,j}(t - \theta_i) \)
\( i = 1, \ldots, 5 \)
\( j = 1, \ldots, 10 \)
Open challenges in hydraulic fracturing

- **Hydraulic fracturing modeling in unconventional reservoirs**
  - Shadow effects by multiple fractures
  - Proppant transport in complex fracture networks and proppant distribution
  - Multiscale of hydraulic fractures

- **Zipper fracturing can increase the complexity of the stimulation to provide enough surface area**
Open challenges in hydraulic fracturing

- The production rate decreases significantly after one year
  - Flow control of the porous medium equation (Hasan et. al, 2012)

References

References

Thank you