Petroleum production optimization
a static or dynamic problem?

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Outline

• Petroleum production optimization
• Formulating the problem and solution strategies
• Production optimization in conventional wells
• Production optimization in shale gas systems
  • Energy policy implications
• Conclusions
Petroleum production systems
Decision hierarchy – time separation

- Investment strategy, choice of technology, drainage strategy, …
- Production and injection targets, well location and completion design, …
- Choke settings, artificial lift settings, separator pressure, well routings, flow assurance, performance monitoring, …
- Topside controls (pressure, level, rate, …), slug control, …
System architecture for typical offshore system
Formulation based on a directed graph

Table 1: Utility sets

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mathbf{N})</td>
<td>Set of nodes (i \in \mathbf{N}).</td>
</tr>
<tr>
<td>(\mathbf{E})</td>
<td>Set of edges (e = (i, j) \in \mathbf{E}), with (i, j \in \mathbf{N}).</td>
</tr>
<tr>
<td>(\mathbf{K})</td>
<td>Timesteps (\mathbf{K} = {0, 1, \ldots, N}). (\mathbf{K}^- = \mathbf{K} \setminus \mathbf{N}).</td>
</tr>
<tr>
<td>(\mathbf{E}^\text{in}_i)</td>
<td>Edges entering node (i), i.e. (\mathbf{E}^\text{in}_i = {e : e = (j, i) \in \mathbf{E}}).</td>
</tr>
<tr>
<td>(\mathbf{E}^\text{out}_i)</td>
<td>Edges leaving node (i), i.e. (\mathbf{E}^\text{out}_i = {e : e = (i, j) \in \mathbf{E}}).</td>
</tr>
<tr>
<td>(\mathbf{E}^\text{snk})</td>
<td>Edges entering a sink node, i.e. (\mathbf{E}^\text{snk} = {e : e = (i, j), \mathbf{E}^\text{out} = \emptyset}).</td>
</tr>
<tr>
<td>(\mathbf{E}^\text{d})</td>
<td>Set of discrete edges, i.e. (\mathbf{E}^\text{d} \subset \mathbf{E}).</td>
</tr>
<tr>
<td>(\mathbf{N}^\text{d})</td>
<td>Nodes with discrete leaving edges, i.e. (\mathbf{N}^\text{d} = {i : i \in \mathbf{N}, \mathbf{E}^\text{out}_i \subset \mathbf{E}^\text{d}} \subset \mathbf{N}).</td>
</tr>
<tr>
<td>(\mathbf{R})</td>
<td>Set of phases - oil, gas and water {oil, gas, wat}.</td>
</tr>
</tbody>
</table>

Table 2: Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_{ik})</td>
<td>Pressure in node (i \in \mathbf{N}) at time (k \in \mathbf{K}).</td>
</tr>
<tr>
<td>(q_{rek})</td>
<td>Flow rate of phase (r \in \mathbf{R}) on edge (e \in \mathbf{E}) at time (k \in \mathbf{K}).</td>
</tr>
<tr>
<td>(Y_{ek})</td>
<td>Boolean variable associated with edge (e \in \mathbf{E}^\text{d}) at time (k \in \mathbf{K}).</td>
</tr>
</tbody>
</table>

The edge may be open \((Y_{ek} = \text{True})\) or closed \((Y_{ek} = \text{False})\). |

maximize \(z = \sum_{r \in \mathbf{R}} \sum_{e \in \mathbf{E}^\text{snk}} \sum_{k \in \mathbf{K}} g_{rk}(q_{rek})\) (1)

subject to

\(\sum_{e \in \mathbf{E}^\text{in}} q_{rek} = \sum_{e \in \mathbf{E}^\text{out}} q_{rek},\) \(\forall r \in \mathbf{R}, i \in \mathbf{N}^{\text{int}}, k \in \mathbf{K}\) (2)

\(\zeta_{rik}(q_{e,k+1}, q_{ek}, p_{ik}) = 0,\) \(\forall r \in \mathbf{R}, i \in \mathbf{N}^{\text{src}}, k \in \mathbf{K}^-\) (3)

\(q_{e,0} = \text{given},\) \(\forall i \in \mathbf{N}^{\text{src}}\) (4)

\(p_{ik} = \text{const.},\) \(\forall i \in \mathbf{N}^{\text{snk}}, k \in \mathbf{K}\) (5)

\(p_{ik} - p_{jk} = f_{e}(q_{ek}, p_{ik}),\) \(\forall e \in \mathbf{E} \setminus \mathbf{E}^\text{d}, k \in \mathbf{K}\) (6)

\(Y_{ek} \implies p_{ik} - p_{jk} = f_{e}(q_{ek}, p_{ik}),\) \(\forall e \in \mathbf{E}^\text{d}, k \in \mathbf{K}\) (7)

\(\neg Y_{ek} \implies q_{ek} = 0,\) \(\forall e \in \mathbf{E}^\text{d}, k \in \mathbf{K}\) (8)

\(\left(\bigvee_{e \in \mathbf{E}^\text{out}_i} Y_{ek}\right) \vee \left(\bigwedge_{e \in \mathbf{E}^\text{out}_i} \neg Y_{ek}\right),\) \(\forall i \in \mathbf{N}^\text{d}, k \in \mathbf{K}\) (9)

\(Y_{ek} \in \{\text{True, False}\},\) \(\forall e \in \mathbf{E}^\text{d}, k \in \mathbf{K}\) (10)

\(\sum_{e \in \mathbf{E}^\text{snk}} q_{rek} \leq C_{rk},\) \(\forall r \in \mathbf{R}, k \in \mathbf{K}\) (11)
Formulation

**Objective**

$$z = \sum_{r \in R} \sum_{e \in E^\text{sink}} \sum_{k \in K} g_{rk}(q_{rek})$$  \hspace{1cm} (1)

**Subject to**

- **Mass balances**
  $$\sum_{e \in E^\text{in}} q_{rek} = \sum_{e \in E^\text{out}} q_{rek},$$  \hspace{1cm} (2)

- **Well inflow models**
  $$\zeta_{rik}(q_{e,k+1}, q_{ek}, p_{ik}) = 0, \quad q_{e,0} = \text{given},$$

- **Pipe pressure drop model**
  $$p_{ik} = \text{const.}, \quad p_{ik} - p_{jk} = f_e(q_{ek}, p_{ik}),$$

- **On-off routing valves**
  $$Y_{ek} \Rightarrow p_{ik} - p_{jk} = f_e(q_{ek}, p_{ik}), \quad \neg Y_{ek} \Rightarrow q_{ek} = 0,$$

- **Flow splitting – not permitted**
  $$\left( \bigvee_{e \in E^\text{out}} Y_{ek} \right) \lor \left( \bigwedge_{e \in E^\text{in}} \neg Y_{ek} \right),$$

- **Processing capacity constraints**
  $$\sum_{e \in E^\text{sink}} q_{rek} \leq C_{rk},$$

- Processing capacity constraints

**Processing capacity contraints**

$$\forall r \in \mathbb{R}, i \in \mathbb{N}^\text{int}, k \in \mathbb{K}$$  \hspace{1cm} (2)

$$\forall r \in \mathbb{R}, i \in \mathbb{N}^\text{src}, k \in \mathbb{K}^- \quad \forall i \in \mathbb{N}^\text{src}$$  \hspace{1cm} (3)

$$\forall i \in \mathbb{N}^\text{snk}, k \in \mathbb{K}$$  \hspace{1cm} (5)

$$\forall e \in \mathbb{E} \setminus \mathbb{E}^d, k \in \mathbb{K}$$  \hspace{1cm} (6)

$$\forall e \in \mathbb{E}^d, k \in \mathbb{K}$$  \hspace{1cm} (7)

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$$\forall e \in \mathbb{E}^d, k \in \mathbb{K}$$  \hspace{1cm} (10)

$$\forall r \in \mathbb{R}, k \in \mathbb{K}$$  \hspace{1cm} (11)
Formulation

- Can handle a wide range of topologies
- Fluid components
- Artificial lift technologies
- Subsea components
- Straightforward extension
  - Normal operational constraints
  - Energy balance (temperature)
Solution techniques

- Models are usually embedded in simulators
  - E.g. pressure drop simulator for a pipeline
  - Derivative free method

- Model approximation
  - Piecewise linearization
    MINLP -> MILP
  - Spline-functions
    MINLP -> MINLP

- Decomposition
  - Dantzig-Wolfe decomposition
  - Lagrangian relaxation

\[
\begin{align*}
\text{maximize} & \quad z = \sum_{r \in \mathbb{R}} \sum_{e \in E^\text{sink}} \sum_{k \in K} g_{rek}(q_{rek}) \\
\text{subject to} & \quad \sum_{e \in E_i^{\text{in}}} q_{rek} = \sum_{e \in E_i^{\text{out}}} q_{rek}, \quad \forall r \in \mathbb{R}, i \in \mathbb{N}^{\text{int}}, k \in K \\
& \quad \zeta_{rik}(q_{e,k+1}, q_{ek}, p_{ik}) = 0, \quad \forall r \in \mathbb{R}, i \in \mathbb{N}^{\text{src}}, k \in K^- \\
& \quad q_{e,0} = \text{given}, \quad \forall i \in \mathbb{N}^{\text{src}} \\
& \quad p_{ik} = \text{const.}, \quad \forall i \in \mathbb{N}^{\text{sink}}, k \in K \\
& \quad p_{ik} - p_{jk} = f_e(q_{ek}, p_{ik}), \quad \forall e \in E \setminus E^d, k \in K \\
& \quad Y_{ek} \implies p_{ik} - p_{jk} = f_e(q_{ek}, p_{ik}), \quad \forall e \in E^d, k \in K \\
& \quad -Y_{ek} \implies q_{ek} = 0, \quad \forall e \in E^d, k \in K \\
& \quad \left( \forall \left( e \in E_i^{\text{out}} \right) \right) \vee \left( \bigwedge \left( e \in E_i^{\text{out}} \right) \right), \quad \forall i \in \mathbb{N}^d, k \in K \\
& \quad Y_{ek} \in \{ \text{True, False} \}, \quad \forall e \in E^d, k \in K \\
& \quad \sum_{e \in E^\text{sink}} q_{rek} \leq C_{rk}, \quad \forall r \in \mathbb{R}, k \in K 
\end{align*}
\]
Static formulation

• Static approximation typically suffices due to slow reservoir dynamics
  - Statoil Troll case
  - Petrobras case - Urucu
  - BP case – West Africa
Static formulation – Is there a business case?

• Backdrop
  • Complex and varying bottleneck structure – in particular in mature assets

• Main reasons for revenue increase
  • Production teams obtain more precise advice, thus they make better operational decisions
  • Faster response to abnormal situations
  • Consistency in operations between teams

Revenue increase does not require any significant CAPEX
Dynamic formulation – When?

• Shale gas wells
  • Much faster well dynamics than conventional reservoirs
  • Have the unique ability to quickly recover loss of production from well shut-ins
Production optimization in shale gas systems

• **Dynamic well scheduling**
  • Developed a shut-in strategy for shale gas systems
  • Use dynamic well inflow models which are tuned towards the dominating dynamics
    Knudsen, Foss (2015), “Designing shale-well proxy models for field development and production optimization problems”, *Journal of Natural Gas Science and Engineering*
Case study - Shale gas in a dynamic energy grid

• Current situation
  • Constant gas supply with underground storage
  • Dynamic gas demand

• Shale gas wells may act as a proxy for underground storage to support dynamic demand directly

Case study

• Electrical utility company operates
  • Natural gas power plant (NGPP)
  • Renewables power sources

• Varying NGPP power demand

• Objective function
  • High penalty if demand is not met
  • Surplus gas sold on spot market

• Receding horizon strategy
  • Schedule shut-ins every 3 hr based on a demand curve and 3 day prediction horizon
Case study

- Electrical utility company operates
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- Varying NGPP power demand
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Shale wells as proxy storage

• The proxy concept supports demand-driven gas production and thus can support utilization of renewables.
• Reduces demand for underground storage and thus increases energy efficiency.
• Provides economic incentives for both shale gas producer and electric utility company.


• Reduces the risk for methane discharge as the massive Aliso Canyon gas leak.

Conclusions

• A static formulation suffices in many relevant cases, in particular for conventional reservoirs

• In some cases reservoir dynamics are fast – mid-life shale wells and oil rims are two important cases. Then a dynamic formulation is of interest

• Dynamic scheduling of shale gas wells has some interesting properties from an energy policy point of view

Additional comments:

• A dynamic formulation is of interest also for conventional reservoirs on a shorter time scale, in particular related to start-up of wells. However, the complexity increase is significant

• Online model calibration and uncertainty are key issues which has not been touched upon explicitly in this presentation